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Empirical Analysis of Stock Return Distribution's Impact upon Market Volatility: Experiences from Australia

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Stock market volatility reflects macroeconomic changes, participants' expectation and interaction. Market volatility research has been conducted extensively. However, different backgrounds, assumptions and methodologies have led to heterogeneous models, making volatility forecasting a controversial topic. This paper tests whether stock return distribution's assumptions influence the performance of volatility forecasting. The methodologies involve empirical analysis using GARCH-Normal, GARCH-Student-t and GARCH-Skewed Generalized Error Distributions. Not only daily returns, realized weekly and monthly volatilities of S&P/ASX 200 Index and ASX All Ordinaries Index are calculated over 10 years, but also the out-of sample-volatilities are compared. The output indicates that GARCH-Student-t is superior to others over short-run forecast horizon while GARCH-SGED performs better than others over long-run forecast horizon. The recommended policies are: high-dimensional volatility modeling and out-of-sample forecasting should be based on appropriate assumptions of stock return distribution to increase accuracy; volatility dependence and volatility-correlation co-movement may reduce the benefits of stock diversification.

1. Introduction

With a relatively strong growth and low inflation economy supported by the cooperative government, robust political and economic institutions, Australian stock markets play a vital role in stimulating the economy, facilitating savings and channeling funds from surplus agents to deficit agents. As the largest Australian stock exchange market, Australian Securities Exchange (ASX) functions as a market operator and the volatility of the stocks listed on the ASX mirrors the uncertainties of the macro-economy and the potential risk underneath the prosperity. Volatility modeling, undoubtedly, is essential to the theory and practice of asset pricing, fund allocation and risk management.

One of the key fundamentals underpinning market volatility modeling lies in the specification of stock return's distribution assumptions, because different backgrounds, assumptions, methodologies and intentions have led to heterogeneous volatility models, making volatility forecasting a controversial topic. Hence, this paper tests the hypothesis of whether stock return's distribution assumptions underlying Australian Securities Exchange's Volatility influence the performance of volatility forecasting.

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The motivation behind the topic is to compare the accuracy of volatility forecasting using GARCH-Normal (GARCH-N) stock return distribution, GARC-Student-t (GARCH-ST) stock return distribution and GARCH-Skewed Generalized Error (GARCH-SGED) stock return distribution so as to identify the most appropriate stock return volatility assumption in different market volatility modeling scenarios.

Previous research generally selects the stock return distribution assumption before modeling market volatility based on some presumed selection criteria. However, this paper differs from the existing literature in the sense that its selection criteria for distribution assumption lies in the forecasting performance outcome of different model specifications and it applies the GARCH models to simulate the ASX stock return dynamics both theoretically and empirically.

This paper is organized as the following. In the first section, the distributions of stock returns are examined, three GARCH sub-models in terms of GARCH-Normal, GARCH-Student-t and GARCH-Skewed Generalized Error Distribution are tested to see if they fit the data drawn from the high-frequency data of S&P/ASX 200 and ASX All Ordinaries Indexes over a ten-year period. In the second section, based on the analysis of these candidate models' static and dynamic forecast performance, suggestions are made to shed light on the choice of data distribution according to various scenarios. Then, under the chosen data distributions, their corresponding logarithmic standard deviations and correlations are formulated to identify the existence of temporal inter-dependence and long memory processes. In the last section, the impacts of the co-movement between volatilities and correlations upon portfolio diversification, risk management and asset pricing are examined to enlighten policy makers.

2. Literature Review

History reveals that stock returns are characterized by volatility clustering, persistence and time variation, the GARCH model has been viewed as the foundation of financial series' volatility analysis; and the skewed distributions of stock returns are commonly observed in reality. The following are previous work in evaluating stock price volatility via the GARCH model under the skewed-GED distribution. Miller and Peng (2006) verified that GARCH-SGED models showed much better volatility predictive ability to GARCH-N model for all forecast horizons. Lee, Su and Liu (2008) showed that the GARCH-SGED model evaluated stock index performance more precisely than the GARCH-N and GARCH-ST models for both high and low levels based on empirical analysis. Lee and Pai (2010) conveyed that the stock market volatility forecasts using the GARCH-Skewed GED model are more accurate than those generated via the GARCH-N and GARCH-ST models in all cases, and they confirmed the significant influences of both skewness and tail-thickness on the conditional distribution of returns,

The importance of ASX's volatility study has been debated heatedly among scholars. Aitken, Garvey and Swan (1995) indicated that the volatility of ASX traded stock prices reflects the efficiency of the stock market's trading mechanism and the speed of absorbing information. Mian and Adam (2001) elucidated that an understanding of the patterns of the volatility dynamics of the listed Australian equities enabled investors to seize profit opportunities. Bertram (2004) revealed that the investigation into the volatility study of the ASX listed equity data helps to capture the behavior of distribution tails and the existence of long memory. Pham, Kalev, Liua and Jarnecicc (2004) summarized that controlling for the effects of trading volume and opening volatility, the conditional variance of ASX stock returns are correlated with the arrival rate of the selected news variable positively and significantly, besides, the inclusion of the news variable in the conditional variance equation of the generalized autoregressive conditional heteroskedastic model reduced the intraday data's volatility persistence. Cheng, Forde and Yang (2007) stated that the volatility of ASX traded stock prices generated hints on the new information release's impacts upon the interaction between buyers and sellers. Hence, a comprehensive research into the volatility dynamics of the ASX listed equities over the period will shed light on risk management.

The research into the volatility dynamics of the ASX equities have been conducted from different perspectives using various methodologies among academics, and the method of the Generalized Auto-Regression Conditional Heteroskedasticity (GARCH) and its variations prove to be widely used among the researchers. Mian and Adam (2001) examined the volatility behavior of intraday high frequency returns of ASX listed stocks and discovered that the volatility of ASX equities followed an L-shaped curve, and they took account of the intraday deterministic volatility seasonal and applied GARCH model to detect the volatility clustering for high frequency stock returns. Hautsch and Jeleskovic (2008) released that price volatilities, trading volume, trading intensities, bid-ask spreads and market depth displayed positive values and persistently clustering over time through analyzing trading behavior and modeling trading processes based on high frequency financial data. Worthington and Higgs (2009) employed the GARCH and asymmetric GARCH (alternatively Threshold ARCH) to interpret the data of ASX stock returns, trading volumes and bid-ask spreads, they not only identified the strong persistency in volatility for the stocks with day-of-week effects and contemporaneous lagged volume of trade, but also concluded that all volatility processes exhibit mean-reverting feature regardless of among the selected stocks in terms of the irregular arrival of new information in generating GARCH effects and the degree of persistence.

The majority of researchers formulated the GARCH model assuming the stock returns followed normal distribution and the appropriateness of normal distribution assumption remains to be tested. Mandelbrot (1963), Fama (1965) and Theodossiou (1998) identified that logarithmic returns of financial assets exhibited skewed leptokurtic distribution rather than normal distribution in most cases. Fergusson and Platen (2005) adopted a wide range of statistical techniques to clarify that the assumption on normality for log-returns of stocks should be strongly rejected due to frequency observation of large excess kurtosis, and they supported leptokurtic log-return densities which exhibited heavy tails and more peaked than that of a Gaussian distribution. Egan (2007) pointed out that the normal distribution fits poorly to the daily percentage stock returns and that the log-normal distribution did not match the single period continuously compounded stock returns. All the above evidence suggests that a parametric distribution that accounts for both skewness (asymmetry) and leptokurtosis (fat tails and peakedness) should be chosen to approximate the stochastic behavior of logarithmic stock returns of financial time series data.

Within all the distributions with skewness and leptokurtosis, the skewed Generalized Error Distribution (skewed GED) is broadly recognized as the proxy distribution for the log returns of the ASX traded stocks among researchers. Theodossiou and Trigeorgis (2003) advocated the skewed GED rather than other families of distributions with skewness and kurtosis such as skewed Student-t distribution; because the main interest lied in the whole distribution rather than the tail part and skewed Student-t distribution is only a good approximation for the distribution of the tail values. Wilhelmsson (2006) investigated the forecasting performance of GARCH model in stimulating stock return dynamics with a collection of different distributions, and he found that skewed GED assumption displayed the most satisfactory fitness for modeling daily, weekly as well as monthly frequency data and contributed most significantly in variance forecasting. Liu, Lee and Lee (2009) examined how specification of return distributions influenced the performance of volatility forecasting via a collection of GARCH models with different return distribution assumptions, and they drew the conclusion that GARCH-Skewed GED was superior to other GARCH models in forecasting stock market volatility based on the mean squared error (MSE) model selection criteria.

However, the majority of researchers models stock return volatility using presumed return distribution assumptions and do not verify if the model forecasting ability changes using other distribution assumptions. Hence, this paper will fill in this research gap by simulating ASX stock return dynamics using a series of distribution assumptions and test the hypothesis of whether the model forecasting performance changes in accordance with different underlying stock distribution assumptions.

3. Data and Methodologies

3.1. Theoretically Modeling

Denote $r_t = \mathrm{Ln}\left(\frac{S_t}{S_{t-1}}\right) \cdot 100$, where r_t is the continuously compounded daily returns of the asset at time t, S_t and S_{t-1} are the spot price of the asset at time t and t-1 respectively, Ω_{t-1} is the information set up to time t-1. Below is the GARCH (p,q) model.

$$r_t = \mu + \rho \cdot r_{t-1} + \varepsilon_t, \quad \varepsilon_t = \sigma_t \cdot z_t, \quad z_t|\Omega_{t-1} \sim f\,(0,1)$$

$$\sigma_t^2 = \alpha + \beta_1 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot \varepsilon_{t-2}^2 + \beta_3 \cdot \varepsilon_{t-3}^2 + \cdots + \beta_p \cdot \varepsilon_{t-p}^2 + \gamma_1 \cdot \sigma_{t-1}^2 + \gamma_2 \cdot \sigma_{t-2}^2 + \gamma_3 \cdot \sigma_{t-3}^2 + \cdots + \gamma_q \cdot \sigma_{t-q}^2$$

Where μ is the intercept parameter and ρ is the lag parameter for the return equation; α is the intercept parameter, $\beta_1, \beta_2, \beta_3 \cdots \beta_p$ are the ARCH coefficients and $\gamma_1, \gamma_2, \gamma_3 \cdots \gamma_q$ are the GARCH coefficients for the volatility equation; ε_t is the innovative disturbance; f (0,1) is the density function with mean 0 and variance 1; α, $\beta_1, \beta_2, \beta_3 \cdots \beta_p$, γ_1, γ_2, $\gamma_3 \cdots \gamma_q$ are non-negative parameters; $\beta_1 + \beta_2 + \beta_3 \cdots + \beta_p + \gamma_1 + \gamma_2 + \gamma_3 + \cdots + \gamma_q < 1$ so as to ensure the conditional variance is positive and stationary. The following discusses three conditional distributions of the error term z_t: (I) standard normal distribution; (II) Student-t distribution; (III) skewed-generalized error distribution (SGED). The maximum likelihood method is used to estimate the parameters in GARCH (p, q) with different distribution specifications using the sample of size n.

3.1.1. GARCH (p, q) Model with Normal Distribution

The density function specification is $f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{{z_t}^2}{2}}$ with the log-likelihood function:

$$\log\left(\left(f(z_t)\right)_{\mathrm{Normal}}\right) = \log\left(f(\mu, \rho, \alpha, \beta_1, \beta_2, \beta_3 \cdots \beta_p, \gamma_1, \gamma_2, \gamma_3 \ \cdots \gamma_q)\right) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{n} \frac{{\varepsilon_t}^2}{{\sigma_t}^2} - \frac{1}{2}\sum_{t=1}^{n} {\sigma_t}^2$$

Where $\sigma_t^2 = \alpha + \beta_1 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot \varepsilon_{t-2}^2 + \beta_3 \cdot \varepsilon_{t-3}^2 + \cdots + \beta_p \cdot \varepsilon_{t-p}^2 + \gamma_1 \cdot \sigma_{t-1}^2 + \gamma_2 \cdot \sigma_{t-2}^2 + \gamma_3 \cdot \sigma_{t-3}^2 + \cdots + \gamma_q \cdot \sigma_{t-q}^2$

3.1.2. GARCH (p, q) Model with Student-t Distribution

I used Student-t distribution advocated by Ardia and Hoogerheide (2010). The density function specification is:

$$f(z_t) = \varepsilon_t \sqrt{\left(\frac{v-2}{v}\right) \cdot w_t \cdot {\sigma_t}^2} \qquad \varepsilon_t \sim \mathrm{N.I.I.D.}(0,1) \qquad w_t \sim \mathrm{IG}\left(\frac{v}{2}, \frac{v}{2}\right)$$

$$\sigma_t^2 = \alpha + \beta_1 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot \varepsilon_{t-2}^2 + \beta_3 \cdot \varepsilon_{t-3}^2 + \cdots + \beta_p \cdot \varepsilon_{t-p}^2 + \gamma_1 \cdot \sigma_{t-1}^2 + \gamma_2 \cdot \sigma_{t-2}^2 + \gamma_3 \cdot \sigma_{t-3}^2 + \cdots + \gamma_q \cdot \sigma_{t-q}^2$$

where $t=1,2,3\cdots n$; $\alpha, \beta_1, \beta_2, \beta_3 \cdots \beta_p, \gamma_1, \gamma_2, \gamma_3 \cdots \gamma_q$ are positive; $N(0,1)$ stands for standard normal distribution; IG stands for inverted gamma distribution; N.I.I.D. stands for normally identically independently distributed; v stands for degrees of freedom and is positive to ensure finite conditional variance.

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{pmatrix}, \qquad w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_n \end{pmatrix}, \qquad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_p \end{pmatrix}, \qquad \gamma = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \\ \gamma_q \end{pmatrix}$$

And the $n \times n$ diagonal matrix and its associated log-likelihood function:

$$\sum(\alpha, w, \beta, \gamma) = \mathrm{diag}\left(\left\{w_t \cdot \frac{v-2}{v} \cdot \left(\sigma_t(\alpha, \beta, \gamma)\right)^2\right\}_{t=1}^{n}\right) \qquad L(\alpha, w, \beta, \gamma \mid \varepsilon) = \frac{1}{\sqrt{|\sum(\alpha,w,\beta,\gamma)|}} e^{-\frac{y'(\sum(\alpha,w,\beta,\gamma))^{-1} y}{2}}$$

3.1.3. GARCH (p, q) Model with standardized Skewed−GED Distribution

I adopted SGED distribution promoted by Theodossin (2000) which allowed the return innovative disturbance to account for both skewness and tail-thickness in the conditional distribution of returns. The density function specification is

$$f(z_t) = \frac{v}{2\theta\tau\left(\frac{1}{v}\right)} \cdot e^{-\frac{|z_t-\delta|^v}{[1-\mathrm{sign}(z_t-\delta)\lambda]^v \theta^v}},$$

Where $\theta = \frac{\sqrt{\tau\left(\frac{1}{v}\right)}}{\sqrt{\tau\left(\frac{3}{v}\right)\left(1+3\lambda^2-\frac{4\left(\tau\left(\frac{2}{v}\right)\right)^2\lambda^2}{\tau\left(\frac{1}{v}\right)\cdot\tau\left(\frac{3}{v}\right)}\right)}}$ and $\delta = \frac{2\lambda\tau\left(\frac{2}{v}\right)}{\sqrt{\tau\left(\frac{1}{v}\right)\tau\left(\frac{3}{v}\right)\left(1+3\lambda^2-\frac{4\left(\tau\left(\frac{2}{v}\right)\right)^2\lambda^2}{\tau\left(\frac{1}{v}\right)\cdot\tau\left(\frac{3}{v}\right)}\right)}}$, λ represents the density function's skewness parameter with -1<λ<1 and v represents the density function's shape parameter which controls for the height and the fat tails of the distribution with v>0. The following is the associated log-likelihood:

$$\log\left(\left(f(z_t)\right)_{\text{Skewed GED}}\right) = \log\left(f(\mu, \rho, \alpha, \beta_1, \beta_2, \beta_3 \cdots \beta_p, \gamma_1, \gamma_2, \gamma_3 \cdots \gamma_q, v, \lambda)\right) =$$

$$n \cdot \ln\left(\frac{v}{2\theta\tau\left(\frac{1}{v}\right)}\right) - \sum_{t=1}^{n}\left(\frac{\left|\frac{\varepsilon_t}{\sigma_t}-\delta\right|^v}{\left[1-\text{sign}\left(\frac{\varepsilon_t}{\sigma_t}-\delta\right)\lambda\right]^v\theta^v} + \ln\sigma_t\right)$$

$$\sigma_t^2 = \alpha + \beta_1 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot \varepsilon_{t-2}^2 + \beta_3 \cdot \varepsilon_{t-3}^2 + \cdots + \beta_p \cdot \varepsilon_{t-p}^2 + \gamma_1 \cdot \sigma_{t-1}^2 + \gamma_2 \cdot \sigma_{t-2}^2 + \gamma_3 \cdot \sigma_{t-3}^2 + \cdots + \gamma_q \cdot \sigma_{t-q}^2$$

3.2. Data Description and Empirical Model Selection

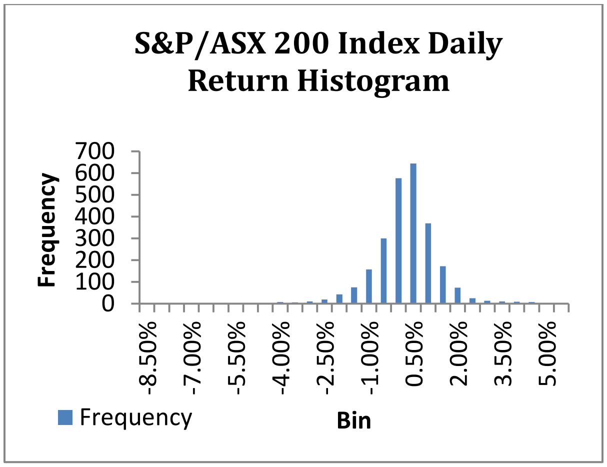
I chose the adjusted close prices of S&P/ASX 200 index and ASX All Ordinaries Index over the period of December 13th, 2001-December 13th, 2011 with daily frequency as the benchmark and indicator of the ASX market. The choice of data length over the 10 years with daily frequency depends on the availability of data and the coverage of the structural changes in Australian Securities Exchange. Below is the data summary table:

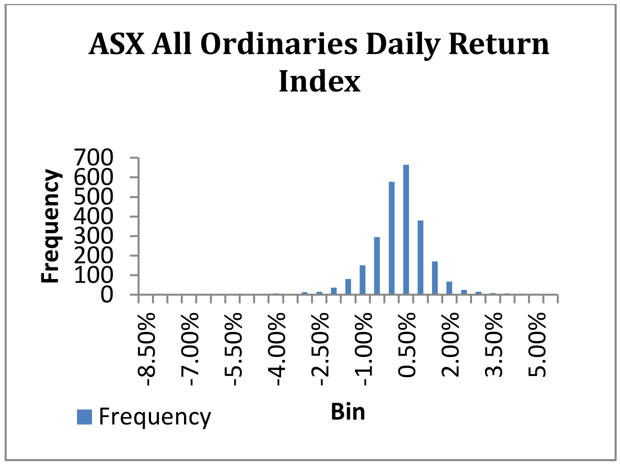
Table 1. Data Set's Statistics Summary

Indicator	S&P/ASX 200 Index	S&P/ASX 200 Index Return	ASX All Ordinaries Index	ASX All Ordinaries Index Return
Mean	4364.5793	0.02%	4367.1573	0.02%
Standard Error	19.3010	0.0002	19.6784	0.0002
Median	4388.6000	0.03%	4399.5500	0.05%
Mode	3171.5000	0.00%	3175.7000	0.00%
Standard Deviation	971.9758	0.0110	990.9790	0.0106
Sample Variance	944736.9481	0.0001	982039.4662	0.0001
Kurtosis	-0.5984	5.3693	-0.6321	5.6841
Skewness	0.4029	-0.3204	0.3775	-0.4203
Range	4128.3000	0.1413	4180.3000	0.1370
Minimum	2700.4000	-8.34%	2673.3000	-8.20%
Maximum	6828.7000	5.79%	6853.6000	5.51%
Observations	2536	2535	2536	2535

Note: Data sources are Australian Bureau of Statistics, Reserve Bank of Australia and Yahoo Finance.

Graph 1. Data Set's Histograms





The distribution of the data indicates that the two time series are both skewed to the left with high peaks. These evidences confirm my suspicion of negative skewness and leptokurtosis underlying the data. However, in order to choose the model based upon scientific principles, I will test the appropriateness of all the three GARCH models in terms of GARCH-N, GARCH-ST and GARCH-SGED on the grounds of a set of criteria.

For the S&P/ASX 200 Index daily returns, I use the GARCH model with the lag structure:

$$\sigma_t^2 = \alpha + \beta_1 \cdot \varepsilon_{t-1}^2 + \gamma_1 \cdot \sigma_{t-1}^2$$

For the ASX All Ordinaries Index daily returns, I use the general GARCH model with the following lag structure:

$$\sigma_t^2 = \alpha + \beta_1 \cdot \varepsilon_{t-1}^2 + \beta_2 \cdot \varepsilon_{t-2}^2 + \beta_3 \cdot \varepsilon_{t-3}^2 + \gamma_1 \cdot \sigma_{t-1}^2 + \gamma_2 \cdot \sigma_{t-2}^2 + \gamma_3 \cdot \sigma_{t-3}^2$$

3.3. Output Estimation and Model Fitness Test

Below is the estimation output of S&P/ASX 200 Index Daily Return output:

Estimation of S&P/ASX 200 Index Daily Return Model with Normal Distribution:

$$\widehat{\text{S\&P/ASX200 Index Return}_t} = 0.0001 - 0.0396\ \text{S\&P/ASX200 Index Return}_{t-1}$$

P Value 0.0000 0.0631

$$\widehat{\sigma_t^2} = 0.0000 + 0.0832\ \varepsilon_{t-1}^2 + 0.9149\ \sigma_{t-1}^2$$

P Value 0.0015 0.0000 0.0000

Estimation of S&P/ASX 200 Index Daily Return Model with Student-t Distribution:

$$\widehat{\text{S\&P/ASX200 Index Return}_t} = 0.0007 - 0.0473\ \text{S\&P/ASX200 Index Return}_{t-1}$$

P Value 0.0000 0.0247

$$\widehat{\sigma_t^2} = 0.0000 + 0.0788\ \varepsilon_{t-1}^2 + 0.9200\ \sigma_{t-1}^2$$

P Value 0.0127 0.0000 0.0000

Estimation of S&P/ASX 200 Index Daily Return Model with SGE Distribution:

$\widehat{\text{S\&P/ASX200 Index Return}}_t = 0.00006 - 0.0396\ \text{S\&P/ASX200 Index Return}_{t-1}$
P Value 0.0000 0.0631
$\widehat{\sigma_t^2} = 0.0000 + 0.0809\ \varepsilon_{t-1}^2 + 0.9188\ \sigma_{t-1}^2$
P Value 0.0226 0.0000 0.0000

For the mean equations, I applied the Correlogram Q-statistics to test the appropriateness of its specification and found that each equation's associated Q-statistics are all insignificant at 5% level of significance, indicating that the mean equations are correctly specified.

For the variance equations, I applied the Ljung-Box Q^2 test in terms of $Q^2(q) = T(T+2)\sum_{i=1}^{q}\frac{\rho(i)}{T-i}$ and found that each equation's associated Q-statistics are all insignificant at 5% level of significance, indicating that the variance equations are correctly specified.

Estimation of ASX All Ordinaries Index Daily Return Model with Normal Distribution:

$\widehat{\text{ASX All Ordinaries Index Return}}_t = 0.0006 - 0.0229\ \text{ASX All Ordinaries Index Return}_{t-1}$
P Value 0.0000 0.2926
$\widehat{\sigma_t^2} = 0.0000 + 0.0933\varepsilon_{t-1}^2 + 0.0592\varepsilon_{t-2}^2 + 0.0916\ \varepsilon_{t-3}^2 + 0.0950\ \sigma_{t-1}^2 - 0.2098\ \sigma_{t-2}^2 + 0.8656\ \sigma_{t-3}^2$
P Value 0.0011 0.0000 0.0000 0.0000 0.0026 0.0000 0.0000

Estimation of ASX All Ordinaries Index Daily Return Model with Student-t Distribution:

$\widehat{\text{ASX All Ordinaries Index Return}}_t = 0.0007 - 0.0313\ \text{ASX All Ordinaries Index Return}_{t-1}$
P Value 0.0000 0.1376
$\widehat{\sigma_t^2} = 0.0000 + 0.0838\varepsilon_{t-1}^2 + 0.0528\varepsilon_{t-2}^2 + 0.0932\ \varepsilon_{t-3}^2 + 0.1210\ \sigma_{t-1}^2 - 0.2056\ \sigma_{t-2}^2 + 0.8516\ \sigma_{t-3}^2$
P Value 0.0100 0.0000 0.0005 0.0000 0.0120 0.0001 0.0000

Estimation of ASX All Ordinaries Index Daily Return Model with Skewed Generalized Error Distribution:

$\widehat{\text{ASX All Ordinaries Index Return}}_t = 0.0007 - 0.0259\ \text{ASX All Ordinaries Index Return}_{t-1}$
P Value 0.0000 0.2223
$\widehat{\sigma_t^2} = 0.0000 + 0.0892\varepsilon_{t-1}^2 + 0.0555\varepsilon_{t-2}^2 + 0.0922\ \varepsilon_{t-3}^2 + 0.1062\ \sigma_{t-1}^2 - 0.2087\ \sigma_{t-2}^2 + 0.8618\ \sigma_{t-3}^2$
P Value 0.0091 0.0000 0.0001 0.0000 0.0088 0.0000 0.0000

For the mean equations, I applied the Correlogram Q-statistics to test the appropriateness of its specification and found that the Q-statistics are all insignificant at 5% level of significance, indicating that the mean equations are correctly specified.

For the variance equations, I applied the Ljung-Box Q^2 test in terms of $Q^2(q) = T(T+2)\sum_{i=1}^{q}\frac{\rho(i)}{T-i}$ and found that each Q-statistics are all insignificant at 5% level of significance, indicating that the variance equations are correctly specified.

3.4. Static Forecast and Dynamic Forecast

Jaggia (2010) suggested that competing models should be compared based on in-sample predictability and out-of-sample forecasting ability.

Table 2. Model Selection Criteria applied to Candidate Models

Model	AIC	SIC	P-Value for the Q Test		
			K=0	K=12	K=18
GARCH-Normal	-6.6398	-6.6283	0.809*	0.772*	0.251*
GARCH-Student-t	-6.6493#	-6.6355	0.532*	0.759*	0.253*
GARCH-Skewed-GED	-6.6474	-6.6359#	0.721*	0.775*	0.256*

Model	AIC	SIC	P-Value for the Q Test		
			K=0	K=12	K=18
GARCH-Normal	-6.7176	-6.6968	0.901*	0.692*	0.330*
GARCH-Student-t	-6.7278#	-6.7048#	0.574*	0.684*	0.325*
GARCH-Skewed-GED	-6.7262	-6.7032	0.781*	0.698*	0.332*

Note: * denotes the residuals are white noises at 5% level of significance. # denotes the best model according to each model selection criteria.

The comparison of in-sample predictability for the estimated models is based on the residuals. In the first step, I used the consistently defined residuals in terms of $\hat{\varepsilon}_t = Y_t - \hat{Y}_t$ to obtain the static forecast. The model selection criteria that I applied here are the Akaike Information Criterion (AIC) in terms of $\mathrm{AIC} = e^{\frac{2k}{n}} \sum_{t=1}^{n} \frac{{e_t}^2}{n}$ and the Bayesian Information Criterion (BIC) in terms of $\mathrm{SIC} = n^{\frac{k}{n}} \sum_{t=1}^{n} \frac{{e_t}^2}{n}$. The AIC and SIC are calculated as sum of squared residuals and are seemed as the penalty for the loss of degree of freedom due to the increased number of parameters used in the model. They both act as the measurement of the information embedded in the residuals. The smaller the values they are, the better the candidate models fit the sample data.

According to the table, for the S&P/ASX 200 Index Daily Return, GARCH-Student-t has the lowest AIC value while GARCH-Skewed-GED has the lowest SIC value; for the ASX All Ordinaries Index Daily Return, GARCH-Student-t has the lowest AIC value while GARH-Skewed-GED has the lowest SIC value. Both AIC and SIC ruled out GARCH-Normal. Hence, the static forecast ruled out the GARCH-Normal model.

Although the AIC and SIC impose penalty upon over-fitting, these criteria are imperfect and may lead to rejection of some appropriate models. Thus, in the second step, out-of-sample forecasting ability is examined via both static and dynamic forecasts.

3.4.1. Static Forecast

Table 3. Out-of-Sample Root Mean Squared Error Statistics for Static Forecast

Forecast Horizon	S&P/ASX 200 Index Daily Return			ASX All Ordinaries Index Daily Return		
	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED
2	0.0124	0.0123#	0.0124	0.0123	0.0122#	0.0123
5	0.0122	0.0119#	0.0119	0.0117	0.0115#	0.0116
10	0.0137	0.01367	0.0137#	0.0130	0.0130	0.0116#
50	0.0145	0.01454	0.0145#	0.0138#	0.0138	0.0138#

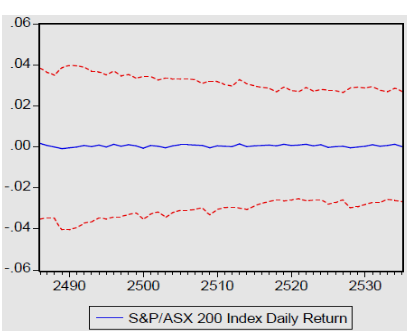
Table 4. Out-of-Sample Mean Absolute Error Statistics for Static Forecast

Forecast Horizon	S&P/ASX 200 Index Daily Return			ASX All Ordinaries Index Daily Return		
	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED
2	0.0123	0.0122#	0.0123	0.0122	0.0120#	0.0121
5	0.0105	0.0105#	0.0105	0.0103	0.0102#	0.0103
10	0.0137	0.0137	0.0137#	0.01138	0.0114#	0.0114
50	0.0119	0.0121	0.0120#	0.0114#	0.0114	0.0114#

Note: * # denotes the best model according to each model selection criteria.

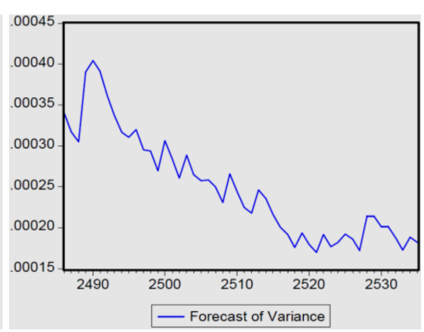
Liu, Lee and Lee (2009) indicated that mean squared errors would raise the return innovative disturbance to the fourth power and induce the loss functions to be very sensible to outliers. Hence, combine all the outputs, I found that GARCH-Student-t and GARCH-Skewed Generalized Error Distribution generate lower root mean squared error statistics and mean absolute error statistics than GARCH-Normal. For the short-run forecasts, GARCH Student-t generates lower root mean squared error statistics and performs forecast with higher accuracy than GARCH Skewed Generalized Error Distribution; for the long-run forecasts, GARCH Skewed Generalized Error Distribution generate lower root mean squared error statistics and mean absolute error statistics than GARCH Student-t. Thus, the RMSE and MAE select GARCH Student-t Distribution model as the best model for the short-run forecast period and GARCH Generalized Error Distribution model as the best model for the long-run forecast period.

Graph 2. S&P/ASX 200 Index Daily Return Models for 50-days Static Forecast
S&P/ASX 200 Index Daily Return Model with Normal Distribution

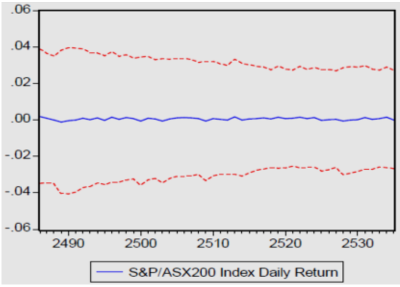


Forecast: S&P/ASX 200 Index Daily Return
Actual: S&P/ASX 200 Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.014535
Mean Absolute Error	0.012042
Mean Abs. Percent Error	105.4044
Theil Inequality Coefficient	0.947492
Bias Proportion	0.004906
Variance Proportion	0.909562
Covariance Proportion	0.085532

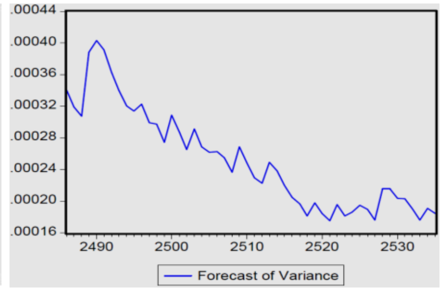


S&P/ASX 200 Index Daily Return Model with Student-t Distribution

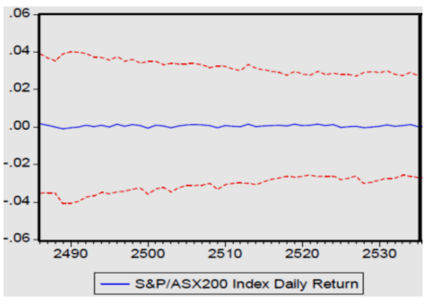


Forecast: S&P/ASX 200 Index Daily Return
Actual: S&P/ASX 200 Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.014544
Mean Absolute Error	0.012050
Mean Abs. Percent Error	106.2575
Theil Inequality Coefficient	0.941161
Bias Proportion	0.004510
Variance Proportion	0.893518
Covariance Proportion	0.101972

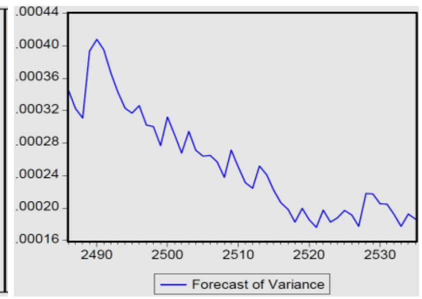


S&P/ASX 200 Index Daily Return with Skewed Generalized Error Distribution

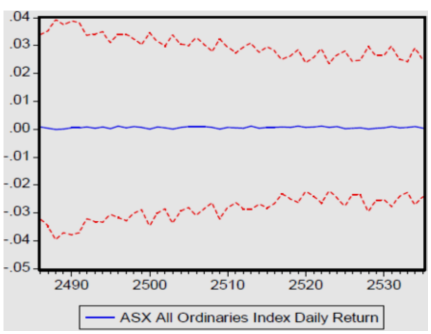


Forecast: S&P/ASX 200 Index Daily Return
Actual: S&P/ASX 200 Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.014535
Mean Absolute Error	0.012044
Mean Abs. Percent Error	105.7609
Theil Inequality Coefficient	0.943910
Bias Proportion	0.004432
Variance Proportion	0.905364
Covariance Proportion	0.090205

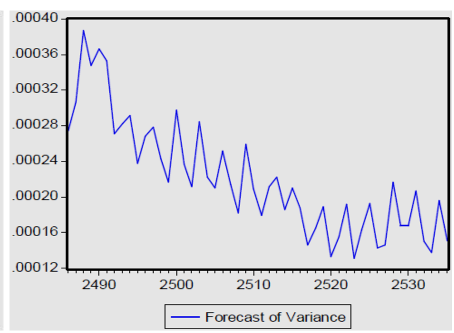


Graph 3. ASX All Ordinaries Index Daily Return for 50-days Static Forecast
ASX All Ordinaries Index Daily Return Model with Normal Distribution

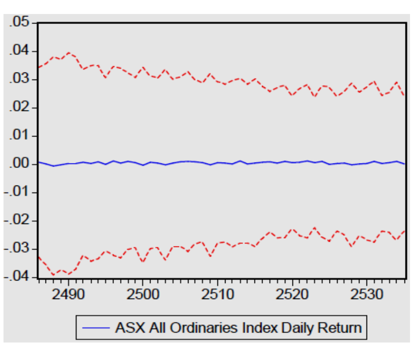


Forecast: ASX All Ordinaries Index Daily Return
Actual: ASX All Ordinaries Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.013763
Mean Absolute Error	0.011390
Mean Abs. Percent Error	102.7449
Theil Inequality Coefficient	0.951411
Bias Proportion	0.006005
Variance Proportion	0.947042
Covariance Proportion	0.046954

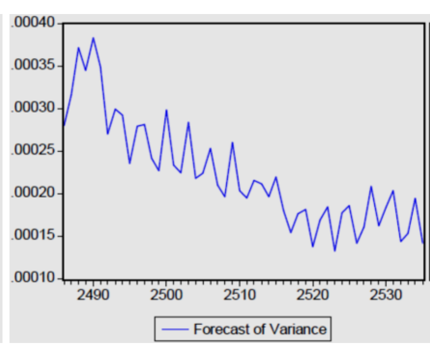


ASX All Ordinaries Index Daily Return Model with Student-t Distribution

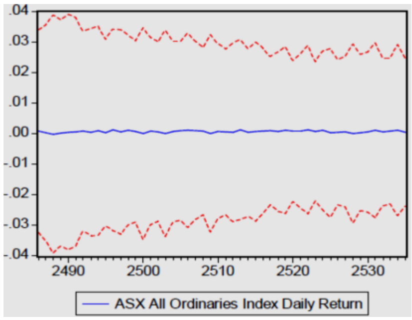


Forecast: ASX All Ordinaries Index Daily Return
Actual: ASX All Ordinaries Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.013769
Mean Absolute Error	0.011396
Mean Abs. Percent Error	103.3415
Theil Inequality Coefficient	0.946086
Bias Proportion	0.005639
Variance Proportion	0.930085
Covariance Proportion	0.064276

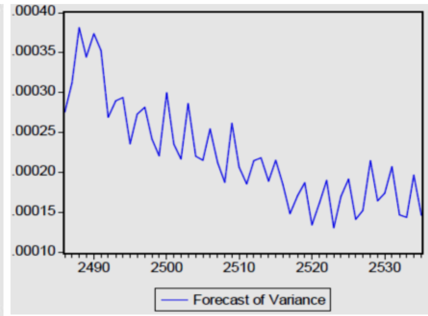


ASX All Ordinaries Index Daily Return with Skewed Generalized Error Distribution



Forecast: ASX All Ordinaries Index Daily Return
Actual: ASX All Ordinaries Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.013763
Mean Absolute Error	0.011391
Mean Abs. Percent Error	103.0216
Theil Inequality Coefficient	0.947820
Bias Proportion	0.005564
Variance Proportion	0.941298
Covariance Proportion	0.053138



3.4.2. Dynamic Forecast

Table 5. Out-of-Sample Root Mean Squared Error Statistics for Dynamic Forecast

Forecast Horizon	S&P/ASX 200 Index Daily Return			ASX All Ordinaries Index Daily Return		
	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED
2	0.012678	0.012618#	0.012666	0.012431	0.012373#	0.012415
5	0.012152	0.012155#	0.012164	0.011681#	0.011683	0.011691
10	0.013656	0.013651	0.013650#	0.012996	0.012993	0.012991#
50	0.014497	0.014497	0.014495#	0.013740	0.013736#	0.013737

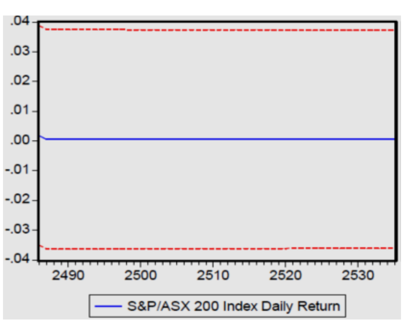
Note: * # denotes the best model according to each model selection criteria.

Table 6. Out-of-Sample Mean Absolute Error Statistics for Dynamic Forecast

Forecast Horizon	S&P/ASX 200 Index Daily Return			ASX All Ordinaries Index Daily Return		
	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED	GARCH-Normal	GARCH-Student-t	GARCH-Skewed GED
2	0.012513	0.012434#	0.012490	0.012242	0.012162#	0.012213
5	0.010778#	0.010763	0.010781	0.010418	0.010401#	0.010418
10	0.011934	0.011932	0.011926#	0.011409	0.011408	0.011403#
50	0.012021#	0.012026	0.012023	0.011366#	0.011366#	0.011366#

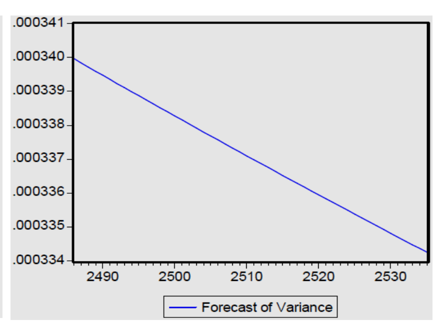
Note: * # denotes the best model according to each model selection criteria.

Graph 4. S&P/ASX 200 Index Daily Return Models for 50-days Dynamic Forecast
S&P/ASX 200 Index Daily Return Model with Normal Distribution

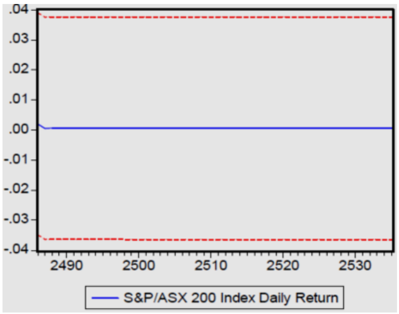


Forecast: S&P/ASX 200 Index Daily Return
Actual: S&P/ASX 200 Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.014497
Mean Absolute Error	0.012021
Mean Abs. Percent Error	102.0809
Theil Inequality Coefficient	0.956669
Bias Proportion	0.004459
Variance Proportion	0.972067
Covariance Proportion	0.023474

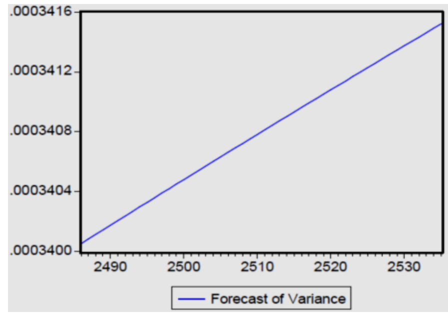


S&P/ASX 200 Index Daily Return Model with Student-t Distribution

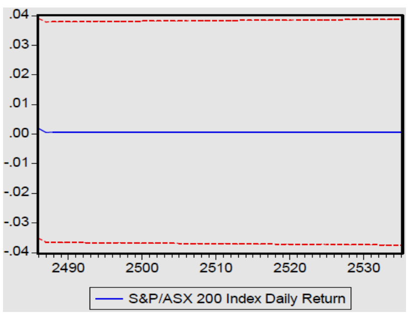


Forecast: S&P/ASX 200 Index Daily Return
Actual: S&P/ASX 200 Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.014497
Mean Absolute Error	0.012026
Mean Abs. Percent Error	102.2859
Theil Inequality Coefficient	0.953187
Bias Proportion	0.004022
Variance Proportion	0.967882
Covariance Proportion	0.028097

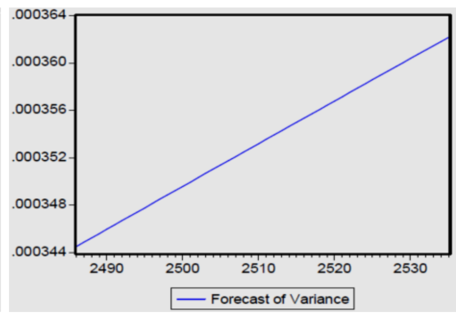


S&P/ASX 200 Index Daily Return with Skewed Generalised ErrorDistribution

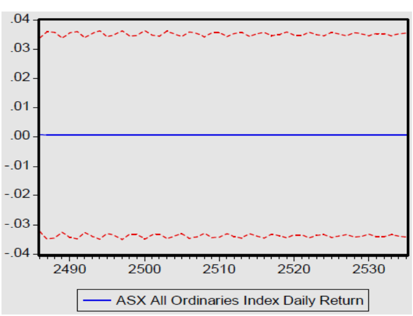


Forecast: S&P/ASX 200 Index Daily Return
Actual: S&P/ASX 200 Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.014495
Mean Absolute Error	0.012023
Mean Abs. Percent Error	102.2507
Theil Inequality Coefficient	0.953256
Bias Proportion	0.004003
Variance Proportion	0.971182
Covariance Proportion	0.024815

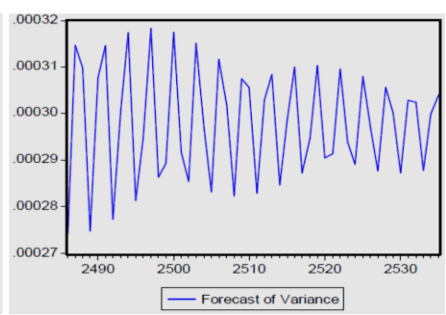


Graph 5. ASX All Ordinaries Index Daily Return for 50-days Dynamic Forecast
ASX All Ordinaries Index Daily Return Model with Normal Distribution

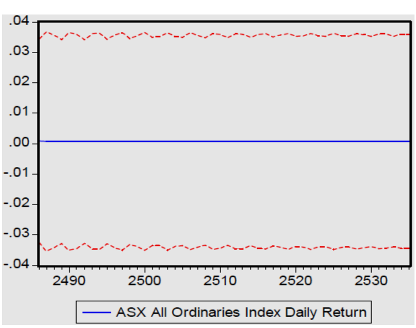


Forecast: ASX All Ordinaries Index Daily Return
Actual: ASX All Ordinaries Index Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.013740
Mean Absolute Error	0.011368
Mean Abs. Percent Error	101.3733
Theil Inequality Coefficient	0.952906
Bias Proportion	0.005687
Variance Proportion	0.991503
Covariance Proportion	0.002809

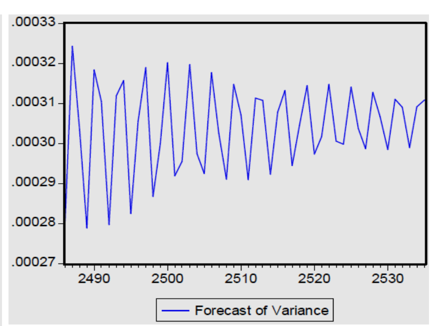


ASX All Ordinaries Index Daily Return Model with Student-t Distribution

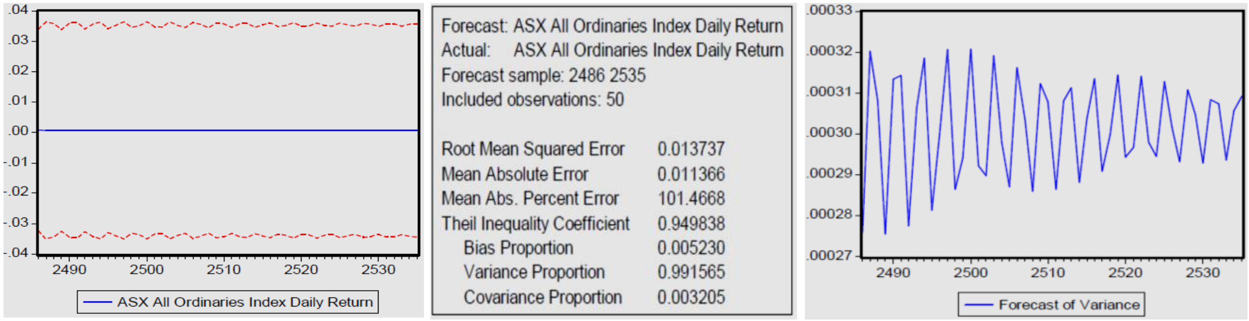


Forecast: ASX All Ordinaries Daily Return
Actual: ASX All Ordinaries Daily Return
Forecast sample: 2486 2535
Included observations: 50

Root Mean Squared Error	0.013736
Mean Absolute Error	0.011366
Mean Abs. Percent Error	101.4594
Theil Inequality Coefficient	0.949804
Bias Proportion	0.005234
Variance Proportion	0.990877
Covariance Proportion	0.003889



ASX All Ordinaries Index Daily Return with Skewed Generalized Error Distribution



3.5. Evaluation of Volatility Forecasting Performance

3.5.1. Loss Function

This paper selected the squared returns r_t^2 as a proxy for the ex post variance, which was suggested in the works of Sadorsky (2006). The forecasting performance of candidate models is evaluated via the standard criteria in terms of Mean Square Error (MSE) and Mean Absolute Error (MAE) as the following:

$$\mathrm{MSE} = \frac{h}{p} \cdot \sum_{t=1}^{p/h} \left(\sigma^2_{\mathrm{Ex\ Post},t+h} - \sigma^2_{\mathrm{Forecasted},t+h}\right)^2 = \frac{h}{p} \cdot \sum_{t=1}^{p/h} \left(r_t^2 - \sigma^2_{\mathrm{Forecasted},t+h}\right)^2$$

$$\mathrm{MAE} = \frac{h}{p} \cdot \sum_{t=1}^{p/h} \left|\sigma^2_{\mathrm{Ex\ Post},t+h} - \sigma^2_{\mathrm{Forecasted},t+h}\right| = \frac{h}{p} \cdot \sum_{t=1}^{p/h} \left|r_t^2 - \sigma^2_{\mathrm{Forecasted},t+h}\right|$$

Where $\sigma^2_{\mathrm{Ex\ Post},t+h}$ and $\sigma^2_{\mathrm{Forecasted},t+h}$ are the ex post and forecasted variances over the window horizon of length h commences at day t respectively.

3.5.2. Model Specification's Significance Test

The test's null hypothesis is that the candidate models possess equal predicative ability and accuracy under the assumption of zero deviation expectation in terms of $E(\varepsilon^2_{I,t} - \varepsilon^2_{II,t}) = 0$, where E denotes the expectation, $\varepsilon^2_{I,t}$ and $\varepsilon^2_{II,t}$ are forecast errors generated by the model I and model II respectively.

This paper utilizes the DM test statistics recommended by Diebold and Mariano (1995):

$$\mathrm{DM} = \frac{1}{p} \cdot \sum_{t=1}^{p} \left(\varepsilon^2_{I,t} - \varepsilon^2_{II,t}\right) \cdot \sqrt{\frac{p}{\left(\gamma_0 + 2\sum_{k=1}^{h-1} \gamma_k\right)}} \sim N(0,1)$$

Where γ_k represents the kth auto-covariance of $(\varepsilon^2_{I,t} - \varepsilon^2_{II,t})$ and DM follows a standard normal distribution asymptotically.

The Diebold and Mariano test statistics and their associated p-values across different forecast horizons are summarized in Table VII below

Table 7. Diebold and Mariano Test for S&P/ASX 200 Index Daily Return

Forecast Horizon	GARCH Normal vs. GARCH Student-t		GARCH Normal vs. GARCH Skewed Generalized Error Distribution		GARCH Skewed Generalized Error Distribution vs. GARCH Student-t	
	DM-Statistic	P-Value	DM-Statistic	P-Value	DM-Statistic	P-Value
2	2.4629**	0.0138	4756.454***	0.0000	-39.7539***	0.0000
5	8111.234***	0.0000	8111.234***	0.0000	-1.7883*	0.0737
10	3.0571***	0.0022	3.09***	0.0020	2.0654**	0.0389
20	1.702*	0.0888	1.7132*	0.0867	2.199**	0.0279
50	1.8274*	0.0676	2.0064**	0.0448	2.1719**	0.0299
100	1.9471*	0.0515	1.6834*	0.0923	1.9741**	0.0484

Table 8. Diebold and Mariano Test for ASX All Ordinaries Index Daily Return

Forecast Horizon	GARCH Normal vs. GARCH Student-t		GARCH Normal vs. GARCH Skewed Generalized Error Distribution		GARCH Skewed Generalized Error Distribution vs. GARCH Student-t	
	DM-Statistic	P-Value	DM-Statistic	P-Value	DM-Statistic	P-Value
2	2.4534**	0.0142	6284.471***	0.0000	-1.6758*	0.0938
5	8194.957***	0.0000	1.9302*	0.0536	-1.908*	0.0564
10	89.6007 ***	0.0000	3.3739***	0.0007	983.7436***	0.0000
20	2.3283**	0.0199	2.2026**	0.0276	1.7108 *	0.0871
50	1.8135*	0.0698	219.4713**	0.0000	219.4713**	0.0000
100	1.8807*	0.0600	1.9755**	0.0482	1.6758*	0.0938

Note: I. *, **, *** denote significance at 10%, 5% and 1% respectively.
II. The Diebold-Mariano statistics and p values are derived from the Diebold-Mariano tests conducted in R.
III. The null hypothesis of DM-test is that the two models exhibit equal predictive ability.
IV. A significant positive (negative) DM-statistic indicates that the first model is dominated by (dominates) the second model in comparison.

The outputs of S&P/ASX 200 Index and ASX All Ordinaries indicates that the GARCH-SGED models are relatively more accurate in forecasting for the long-term forecast horizon, while the GARCH-Student-t is relatively more accurate in forecasting for the long-term forecast horizon, however, both GARCH-SGED and GARCH-Student-t performance better than GARCH-Normal models. Hence, in order to improve forecast accuracy, for short-term forecast horizon, the asset returns' assumption of student-t distribution should be established; for long-term forecast horizon, the asset returns' assumption of skewed generalized error distribution should be launched.

6. Temporal Dependence, Long Memory and Scaling

6.1. Scaling Effects

Table 9. Unconditional Distribution of Monthly Return and Scaling Effects

Index	Statistics	Return r				Return/Standard Deviation=r/σ			
		Mean	Standard Deviation	Skewness	Kurtosis	Mean	Standard Deviation	Skewness	Kurtosis
S&P/ASX 200 Index	Minimum Return r	-0.15%	0.0113	-1.3423	0.3043	-12.74%	1.1518	0.1369	3.48
	Median Return r	0.14%	0.0057	-0.026	0.525	50.09%	1.1445	-0.1418	3.53
	Maximum Return r	0.33%	0.0052	-0.5464	0.3233	78.58%	1.018	-0.0922	3.04
ASX All Ordinaries	Minimum Return r	0.13%	0.0108	-1.353	0.2118	-7.12%	1.129	-0.061	3.74
	Median Return r	-0.01%	0.0057	-0.1919	0.2553	63.68%	1.2956	-0.0334	3.24
	Maximum Return r	0.17%	0.0051	-0.383	0.1954	46.94%	0.9635	-0.0125	3.01

Note: The table summarizes the monthly return distribution of S&P/ASX 200 Index and ASX All Ordinaries Index. The sampling period is from January 1st, 2001 to December 31st, 2010. The realized monthly volatilities are calculated from the corresponding weekly returns within each investigation month.

The table indicates that the distributions of the monthly returns of S&P/ASX 200 Index and ASX All Ordinaries Index are negatively-skewed and platykurtic. However, after standardization, the returns of the standardized monthly returns of S&P/ASX 200 Index and ASX All Ordinaries Index are distributed approximately normally.

6.2. Logarithmic Standard Deviations

Table 10. Unconditional Distributions of Realized Monthly Variances and Logarithmic Standard Deviations

Index	Statistics	Variance σ_i^2				Log (Standard Deviation) = log (σ)			
		Mean	Standard Deviation	Skewness	Kurtosis	Mean	Standard Deviation	Skewness	Kurtosis
S&P/ASX 200 Index	Minimum Return r	0.01%	0.0001	0.7781	0.1892	-237.46%	0.0749	0.3484	2.37
	Median Return r	0.02%	0.0001	1.7372	3.9359	-188.94%	0.0893	0.4748	3.31
	Maximum Return r	0.05%	0.0004	1.4772	1.5485	-173.27%	0.1817	0.1233	2.53
ASX All Ordinaries	Minimum Return r	0.00%	0.0000	0.6369	0.1046	-239.75%	0.0785	0.3535	2.62
	Median Return r	0.02%	0.0001	0.9546	1.8960	-191.31%	0.1081	0.6439	2.24
	Maximum Return r	0.04%	0.0004	1.3843	1.7922	-175.52%	0.1908	0.1058	2.58

Note: The table summarizes the monthly volatility distributions. The sampling period is from January 1st, 2001 to December 31st, 2010. The realized monthly variances and logarithmic standard deviations are calculated from the corresponding weekly volatilities within each investigation month.

The table reveals that the distributions of the monthly variances of S&P/ASX 200 Index and ASX All Ordinaries Index are positively-skewed and platykurtic. However, the logarithmic standard deviations of the monthly returns of S&P/ASX 200 Index and ASX All Ordinaries Index exhibit approximate normal distributions.

6.3. Co-variances and Correlations

Table 11. Unconditional Monthly Covariance and Correlation Distributions

Index	Statistics	Covariance σ_{ij}^2				Correlation rij			
		Mean	Standard Deviation	Skewness	Kurtosis	Mean	Standard Deviation	Skewness	Kurtosis
S&P/ASX 200 Index	Minimum Return r	-0.0010%	0.00002	-0.69	3.58	-11.24%	0.2629	-0.57	2.67
	Median Return r	0.0010%	0.00002	1.60	4.34	12.77%	0.2554	0.12	σ_{ij}^2 2.95
	Maximum Return r	0.0020%	0.00002	3.22	10.66	9.73%	0.1698	0.28	2.74
ASX All Ordinaries	Minimum Return r	-0.0030%	0.00004	0.15	3.95	-12.09%	0.1574	-0.05	3.41
	Median Return r	-0.0010%	0.00003	1.49	3.17	-4.81%	0.2536	0.32	2.54
	Maximum Return r	0.0020%	0.00003	0.45	3.15	12.10%	0.1865	0.35	2.58

Note: The table summarizes the unique realized monthly co-variance and correlation distributions between S&P/ASX 200 Index and ASX All Ordinaries Index. The sampling period is from January 1st, 2001 to December 31st, 2010. The realized monthly co-variances and correlations are calculated from the corresponding weekly co-variances and correlations within each investigation month.

The table elucidates that the distributions of the realized monthly co-variances between S&P/ASX 200 Index and ASX All Ordinaries Index are heterogeneous across time although the majority exhibits positive skewness and leptokurtosis. However, the realized monthly correlations between S&P/ASX 200 Index and ASX All Ordinaries Index follow approximately normal distributions.

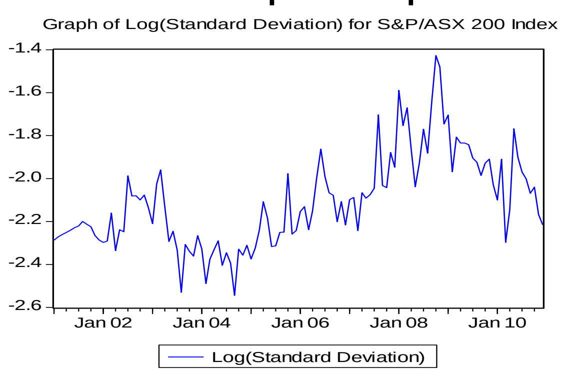
The above analysis captures the features embedded in the stock returns' generating processes. And it reflects that all the distributions of stock volatilities vary considerably across time. Hence, extensive and dynamic risk management is required.

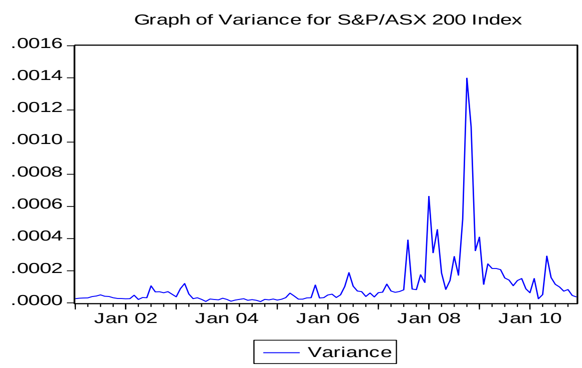
6.4. Dynamic Volatility Dependence

The existence of long-run dependence in co-variances and correlations are examined. The following graphs and tables manifest the dynamic volatility clustering effects.

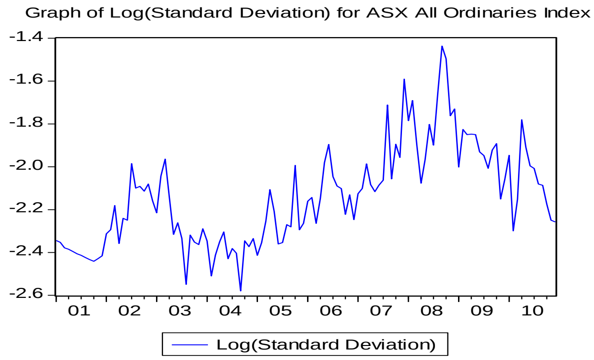
6.4.1. Logarithmic Standard Deviations

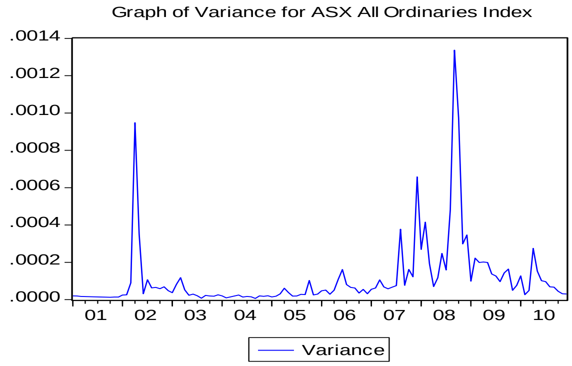
Graph 6. Graphs of Volatilities for S&P/ASX 200 Index





Graph 7. Graphs of Volatilities for ASX All Ordinaries Index





Based on these graphs, it is evident that the two monthly index returns are positively serially correlated with distinguishable high and low volatility periods. Table 12 concludes the results of the significance tests of the phenomenon.

Table 12. Dynamic Volatility Dependence

Index	Log (Standard Deviation) = log (σ)					Correlation r_{ij}				
	Indicator	Q_{22}	ADF Lag Length: 22 (Fixed)	d_{GPH}	d_S	Indicator	Q_{22}	ADF Lag Length: 22 (Fixed)	d_{GPH}	d_S
S&P/ASX 200 Index	Minimum log(σ)	12.359	-0.041	0.753	0.786	Minimum r_{ij}	11.549	-0.04	0.751	0.774
	Median log(σ)	42.359	-0.025	0.845	0.852	Median r_{ij}	23.415	\ -0.038	0.813	0.869
	Maximum log(σ)	51.9	-0.012	0.936	0.967	Maximum r_{ij}	31.364	-0.019	0.897	0.912
ASX All Ordinaries Index	Minimum log(σ)	11.112	-0.039	0.752	0.765	Minimum r_{ij}	10.597	-0.036	0.785	0.811
	Median log(σ)	21.628	-0.02	0.831	0.843	Median r_{ij}	16.378	-0.031	0.791	0.835
	Maximum log(σ)	31.586	-0.013	0.928	0.931	Maximum r_{ij}	21.566	-0.024	0.821	0.892

Note: The table summarizes the dynamic volatility dependence in 120 realized monthly logarithmic standard deviations for the S&P/ASX 200 Index and ASX All Ordinaries Index. The third and eighth columns show the Ljung-Box portmanteau tests for up to 22nd order autocorrelation, Q_{22}. The fourth and ninth columns show the Augmented Dickey-Fuller tests for a unit root with 22 augmentation lags (Assume 30 days per month and 8 non-trading days per month, hence trading days=30-8=22). The fifth and tenth columns show the Geweke-Porter-Hudak test estimates for the degree of fractional integration, d_{GPH}. The sixth and eleventh columns show the scaled estimates for the degree of fractional integration, d_S.

The third and eighth columns summed up the Ljung-Box portmanteau tests for the joint significance of the initial 22 autocorrelations of log (Standard Deviation). It conveys that the null hypothesis of zero autocorrelation is rejected by both indices. The correlograms for the two indices overwhelmingly exceed the Bartlett ninety-five percent confidence band which indicates slow decay rates. Hence, the long-run dependence in the stock market volatility is non-neglectable in risk management. The literature can be extended through stochastic volatility models such as fractional integrated GARCH model.

6.4.2 Volatility-Correlation Co-movement

The parametric estimation of multivariate volatility models is sophisticated due to lack of information about the volatile behaviour among individual stock returns. The eighth and eleventh columns of Table XI displays the statistical summary for the realized correlations between pairs of stocks, the third to the fifth lines are the correlations between stocks which compose the S&P/ASX 200 Index while the sixth to the eighth lines are the correlations between stocks which compose the ASX All Ordinaries Index. The output of Ljung-Box portmanteau statistics for up to 22nd order serial correlation and ADF tests exhibits strong dependence and predictability in these correlations.

7. Conclusions

In the context of a continually changing and reforming financial market, stock market volatility plays a vital role in indicating macroeconomic environment changes, market participants' expectation and interaction mechanism. Market volatility research has been conducted by worldwide academics from different perspectives. However, different backgrounds, assumptions, methodologies and intentions have led to heterogeneous volatility models, making volatility forecasting a controversial topic. This paper concludes that stock return distribution assumptions underlying Australian Securities Exchange's Volatility influence the performance of volatility forecasting. The output indicates that based on the model selection criteria of MSE and MAE, the GARCH-ST is superior to either GARCH-N or GARCVH-ST over short-run forecast horizon while GARCH-SGED performs better than either GARCH-N or GARCH-ST over long-run forecast horizon. Next the DM-tests confirm these conclusions and demonstrate that modeling market volatility should take account of the negative skewness and leptokurtosis embedded in the stock return distributions. Then dynamic volatility dependence and volatility-correlation co-movement are identified and examined to solidify and extend current volatility research. Finally, the following risk management techniques and policies are suggested: high-dimensional volatility modeling and out-of-sample forecasting should be based on appropriate assumptions of stock return distribution to increase forecast accuracy; volatility dependence and volatility-correlation co-movement may reduce the benefits of stock diversification.

References

Adam, C & Mian, G 2001, 'Volatility dynamics in high frequency financial data: an empirical investigation of the Australian equity returns', *Applied Financial Economics* 11, 12.

Aitken, M, Garvey, G & Swan, P 1995, 'How brokers facilitate trade for long term clients in competitive securities markets', *Journal of Business,* 68**,** 33.

Allen, D, Cheng, A, Comerton-Forde, C & Yang, J 2007, 'Returns, Volatility and Liquidity on the ASX: Undisclosed vs. Disclosed Limit Orders', *Finance and Economics & FIMARC Working Paper Series.* Edith Cowan University.

Allen, D, Lim, L & Winduss, T 2007, 'AUSFTA and its implications for the Stock Markets in the Pacific Basin Countries', *Regionalism, Trade and Economic Development in the Asia-Pacific Region.* Cheltenham, UK

Ardia, D & Hoogerheide, L 2010, Bayesian Estimation of the GARCH(1,1) Model with Student-t Innovations, *Tinbergen Institute Discussion Paper.* University of Fribourg, Switzerland and Erasmus University Rotterdam

Betram, W 2004, 'An empirical investigation of Australian Stock Exchange data', *Physica A: Statistical Mechanics and its Applications,* 341, 14.

Egan, W 2007, The Distribution of S&P 500 Index Returns. 15.

Fama, E 1965, 'The Behavior of Stock Market Prices', *The Journal of Business,* 38**,** 72.

Hardle, W, Hautsch, N & Overbeck, L 2006, Applied Quantitative Finance. Springer Berlin Heidelberg

Hautsch, N & Jeleskovic, V 2008, 'High-Frequency Volatility and Liquidity', *Applied Quantitative Finance*.

Jaggia, S 2010, Forecasting with ARMA Models. San Luis Obispo, CA: California Polytechnic State University.

Lee, M, Su, J & Liu, H 2008, 'Value-at-risk in US stock indices with skewed generalized error distribution', *Applied Financial Economics Letters,* 4**,** 7.

Lee, Y & Pai, T 2010, 'REIT volatility prediction for skew-GED distribution of the GARCH model', *Expert Systems with Applications: An International Journal* 37**,** 5.

Liu, H, Lee, Y & Lee, M 2009, 'Forecasting China Stock Markets Volatility via GARCH Models Under Skewed-GED Distribution', *Journal of Money, Investment and Banking*, 11.

Mandelbrot, B 1963, 'Correction of an Error in "The Variation of Certain Speculative Prices', *The Journal of Business,* 45**,** 2.

Miller, N & Peng, L 2006, 'Exploring Metropolitan House Price Volatility', *Journal of Real Estate Finance and Economics,* 33**,** 14.

Pham, P, Kalev, P, Liu, W & Jarnecic, E 2004, 'Public Information Arrival and Volatility of Intraday Stock Returns', *Journal of Banking and Finance,* 28**,** 27.

Theodossiou, P 1998, 'Financial Data and the Skewed Generalized t Distribution', *Management Science,* 44**,** 12.

Theodossiou, P & Trigeorgis, L 2003, 'Option pricing when log-returns are skewed and leptokurtic', *The Eleventh Annual Conference of the Multinational Finance Society.* Istanbul.

Theodossiou, P 2000, Skewed Generalized Error Distribution of Financial Assets and Option Pricing. Cyprus: Cyprus University of Technology

Wilhelmsson, A 2006, 'GARCH Forecasting Performance under Different Distribution Assumptions', *Journal of Forecasting,* 25**,** 18.

Worthington, A & Higgs, A 2009, 'Modeling the Intraday Return Volatility Process in the Australian Equity Market: An Examination of the Role of Information Arrival in S&P/ASX 50 Stocks', *International Business Research,* 1, 8.